Universal set of quantum gates for double-dot spin qubits with fixed interdot coupling (Auxiliary material)

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APPENDIX A: LEAKAGE ERRORS DUE TO ADDITIONAL TRIPLETS

In this section, we justify the omittance of the double-dot triplet states $|T_+\rangle$ and $|T_-\rangle$, with magnetic quantum numbers $S_z = \pm 1$, from our calculations.

First of all, when the inhomogeneous field is exactly along the z-direction (as intended in this proposal), the $S_z = 0$ computational subspace (comprising the singlet and $|T_0\rangle$ states) is completely decoupled from the $S_z = \pm 1$ subspace. In other words, no mixing occurs and therefore no error is introduced.

In a real experiment, it may be challenging to create an inhomogeneous field that is exactly along the z-direction, and therefore a small but finite component of the inhomogeneous field perpendicular to z may exist. This component will lead to a mixing of order $\delta h_\perp^2/E_Z^2$ between the two subspaces, where $\delta h_\perp$ is the component of the inhomogeneous field perpendicular to the z-direction, and E_Z is the Zeeman energy separating the triplet states. The Zeeman energy is given as $E_Z = g\mu_B B_z$, where g is the g-factor, μ_B the Bohr magneton, and B_z the applied magnetic field in z direction. Thus, application of a sufficiently large magnetic field will quench the mixing so that the remaining error is below the desired threshold for quantum error correction.

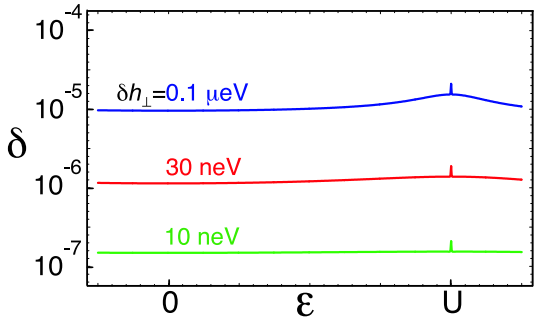

FIG. 5: Leakage error rate δ into the $|T_+\rangle$ and $|T_-\rangle$ states due to a transverse inhomogeneous field gradient $|\delta h_\perp|$ of 10 neV (green), 30 neV (red), 0.1 μeV (blue), for a tunneling matrix element of $t = 5\,\mu$eV and an on-site Coulomb energy $U = 4$ meV, plotted as a function of the misalignment ϵ. The Zeeman energy is chosen to be $E_Z = 10\,\mu$eV corresponding to $B \simeq 400$ mT for GaAs ($g = -0.44$) or $B \simeq 90$ mT for carbon nanotubes ($g = 2$). For the parameters chosen, δ is smaller than 10^{-4} over the full working range.

We now show quantitatively, by extending the Hamiltonian Eq. (1) of the main text, that large values of $\delta h_\perp$ can be tolerated. The extended Hamiltonian in the basis $|(1,1)T_+\rangle$, $|(1,1)T_-\rangle$, $|(1,1)T_0\rangle$, $|(1,1)S\rangle$, $|(0,2)S\rangle$, $|(2,0)S\rangle$ can then be expressed as a 6-by-6 matrix,

$$H = \begin{pmatrix} g\mu_B B & 0 & 0 & -\delta h_\perp^*/2\sqrt{2} & 0 & 0 \\ 0 & -g\mu_B B & 0 & \delta h_\perp/2\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & \delta h/2 & 0 & 0 \\ -\delta h_\perp/2\sqrt{2} & \delta h_\perp^*/2\sqrt{2} & \delta h/2 & 0 & \sqrt{2}t & \sqrt{2}t \\ 0 & 0 & 0 & \sqrt{2}t & U-\varepsilon & 0 \\ 0 & 0 & 0 & \sqrt{2}t & 0 & U+\varepsilon \end{pmatrix}, \tag{A1}$$

where $\delta h = h_{z1} - h_{z2}$ denotes the longitudinal component of the field difference between the two dots. The mixing with the triplet states is due to the combinations $\delta h_\perp = \delta h_x + i\delta h_y$ and $\delta h_\perp^* = \delta h_x - i\delta h_y$ of the transverse field gradient components δh_x and δh_y along x and y, respectively. The mixing-induced error is found by calculating the eigenstates of this Hamiltonian, and squaring the admixture amplitudes.

In Fig. 5, we plot the leakage error probability δ (on a logarithmic scale) as a function of the misalignment between the dots, ϵ, for several values of the transverse field gradient $|\delta h_\perp|$. We take a small static magnetic field such that E_Z=10 μeV, which corresponds to roughly 400 mT for a g-factor of 0.44 (as in GaAs), 90 mT for a g-factor of 2 (as in carbon nanotubes) and 20 mT for materials with a g-factor of 10 (as in some InAs dots).

We see that this small magnetic field is already enough to push the mixing error below 10^{-4} at all values of ϵ, even for a large transverse component of 0.1 μeV. (Note that this transverse component of 0.1 μeV is already larger than the maximum δh along z allowed for a Z-gate error threshold below 10^{-4} for the same parameters, see Fig. 3b of the main text.)

From our anaysis it is apparent that the error due to an undesired transverse component of the inhomogeneous field can easily be suppressed to below a typical error threshold of 10^{-4}, by applying a small magnetic field B along the z-direction. The triplet states $|T_+\rangle$ and $|T_-\rangle$ can thus be safely disregarded.

APPENDIX B: EFFECT OF THE EXCITED QUANTUM DOT ORBITAL

In Eq. (1), only the orbital ground state of each quantum dot has been taken into account, leading to a total of three singlets and three triplets. The inclusion of higher orbitals in the calculation does not change the principle of operation, and also will not introduce additional errors or mixing. The only effect is that the energies of the qubit basis states ($|S\rangle$ and $|T_0\rangle$) will be slightly renormalized, which has a small effect on the range of experimental parameters in which the proposed scheme works. In fact, it turns out that the inclusion of higher orbitals *relaxes* the experimental constraints, and Fig. 3b therefore outlines a conservative, minimum range of suitable parameters.

We demonstrate now quantitatively how the inclusion of the first excited orbital changes the energies. We exclude the $S_z = \pm 1$ triplets (see Sec. A) and are left with one triplet only. The effect of an excited orbital level on the quantum dots is to give rise to two additional triplets $|(0,2)T_0\rangle$ and $|(2,0)T_0\rangle$, split from the $|(0,2)S\rangle$ and $|(2,0)S\rangle$ states by an exchange energy $J(0,2)$ and $J(2,0)$ respectively, as well as to two additional singlets $|(0,2)S'\rangle$ and $|(2,0)S'\rangle$, split from the $|(0,2)T_0\rangle$ and $|(2,0)T_0\rangle$ states by $J'(0,2)$ and $J'(2,0)$. These four states are formed with one electron in the orbital ground state and one in the orbital excited state. We take $J(0,2)$=$J(2,0)$ and $J'(0,2)$=$J'(2,0)$ from here on.

The Hamiltonian including both the additional triplet and singlet due to the excited orbital level can be written in the basis $|(1,1)T_0\rangle$, $|(1,1)S\rangle$, $|(0,2)S'\rangle$, $|(0,2)T_0\rangle$, $|(0,2)S\rangle$, $|(2,0)S'\rangle$, $|(2,0)T_0\rangle$, $|(2,0)S\rangle$, as an 8-by-8 matrix,

$$\begin{pmatrix}
0 & dh/2 & 0 & \sqrt{2}t' & 0 & 0 & \sqrt{2}t' & 0 \\
dh/2 & 0 & \sqrt{2}t' & 0 & \sqrt{2}t & \sqrt{2}t' & 0 & \sqrt{2}t \\
0 & \sqrt{2}t' & U+\epsilon+J(0,2)+J'(0,2) & 0 & 0 & 0 & 0 & 0 \\
\sqrt{2}t' & 0 & 0 & U+\epsilon+J(0,2) & 0 & 0 & 0 & 0 \\
0 & \sqrt{2}t & 0 & 0 & U+\epsilon & 0 & 0 & 0 \\
0 & \sqrt{2}t' & 0 & 0 & 0 & U-\epsilon+J(0,2)+J'(0,2) & 0 & 0 \\
\sqrt{2}t' & 0 & 0 & 0 & 0 & 0 & U-\epsilon+J(0,2) & 0 \\
0 & \sqrt{2}t & 0 & 0 & 0 & 0 & 0 & U-\epsilon
\end{pmatrix}, \tag{B1}$$

where t and t' denote the inter-dot tunneling matrix elements for tunneling between ground states and between a ground and an excited state (typically, $t' \approx t$). We take $t = t' = 5\,\mu$eV in our examples.

The coupling to the single-dot states lowers the energy of both qubit basis states. However, since the single-dot triplet states are closer in energy to the double dot triplet than the single-dot excited singlets to the double dot singlet, the energy of $|T_0\rangle$ will be lowered more than that of $|S\rangle$ in the range of ϵ that we are interested in. Near the avoided crossing ($\epsilon = U$), the energy splitting between $|T_0\rangle$ and $|S\rangle$ is dominated by the tunnel coupling ($J \approx \sqrt{2}t$), and is hardly affected by inclusion of the higher orbitals. At ϵ=0 however, the energy difference between the basis states is reduced. Since we need $\delta h \geq J$ for the X-rotations, a smaller J means that δh can be smaller as well. Thus, the only effect of the inclusion of the higher orbitals is thus that the constraint on δh is slightly relaxed.

By diagonalizing the Hamiltonian, Eq. (B1), one finds that for typical parameters, the contribution from the single-dot triplets $|(0,2)T_0\rangle$ and $|(2,0)T_0\rangle$ is almost cancelled by the $|(0,2)S'\rangle$ and $|(2,0)S'\rangle$ contributions. We plot the tilt angle θ of the qubit rotation axis in Fig. 6, taking into account both the $|(0,2)T_0\rangle$ and $|(2,0)T_0\rangle$ triplets and the S' singlets. Comparing this result to Fig. 3a in the article, it is evident that there is only a small correction due to the excited quantum dot orbital ($< 20\%$). For larger U and/or smaller $J'(0,2)$, the effect will be even smaller. In conclusion, the error analysis (Fig. 3b in the article) is actually a bit too negative, and Fig. 3b therefore outlines a *minimum* suitable range of parameters.

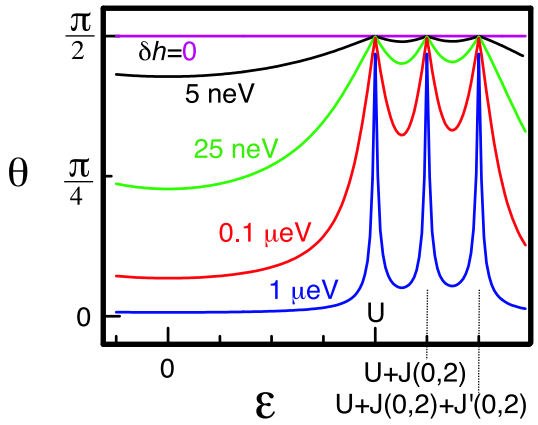

FIG. 6: Tilt angle θ as a function of the misalignment ϵ, including the single-dot triplet and excited singlet states, for $\delta h = 0$, 5 neV, 25 neV, 0.1 μeV, and 1 μeV. Here, U=4 meV, $J(0,2)$=1 meV and $J'(0,2)$ is also 1 meV.